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Backward Compton Scattering and QED with Noncommutative Plane in the Strong Uniform Magnetic Field

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Abstract

In the strong uniform magnetic field, the noncommutative plane (NCP) caused by the lowest Landau level (LLL) effect, and QED with NCP (QED-NCP) are studied. Being similar to the condensed matter theory of quantum Hall effect, an effective filling factor $f(B)$ is introduced to characterize the possibility that the electrons stay on the LLL. The analytic and numerical results of the differential cross section for the process of backward Compton scattering in accelerator with unpolarized or polarized initial photons are calculated. The existing data of BL38B2 in Spring-8 have been analyzed roughly and compared with the numerical predictions primitively. We propose a precise measurement of the differential cross sections of backward Compton scattering in a strong perpendicular magnetic field, which may reveal the effects of NCP.

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Key Words: Compton scattering in magnetic field; noncommutative QED; electron-beam in accelerator; lowest Landau level.

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I. INTRODUCTION

The physics related to the lowest Landau level (LLL) and corresponding spacetime noncommutativity, especially noncommutative field theory (NCFT), have long been studied with considerable interest [1], and appear naturally in fundamental field theory [2, 3] and condensed matter theory [4, 5]. Spacetime noncommutativity was proposed by Heisenberg in the 1930's, in order to introduce an effective ultraviolet cutoff to control the ultraviolet divergences in quantum field theory. Peierls applied it to non-relativistic electronic systems in external magnetic fields, which is the first phenomenological realization of spacetime noncommutativity, and Snyder published it with systematic analysis in 1947 [1]. Recently, noncommutative QED (NCQED) [2] and other NCFTs have been constructed as limits of string/M theory [1], and as the LLL approximation of QED or the Nambu-Jona-Lasinio model in the strong magnetic field [3]. In condensed matter theory, NCFT, particularly the noncommutative Chern-Simons theory [4], provides a better mean field theory description of the fractional quantum Hall states, which can reproduce the detailed properties and the correct quantitative features of quasiparticles. In the present paper, we try to explore the effect of space noncommutativity caused by the LLL, and the possibility to measure it by considering backward Compton scattering in the external magnetic field in accelerator.

Considering a non-relativistic electron in a uniform magnetic field [6],

$$L = \frac{1}{2}m_e(\dot{x}^2 + \dot{y}^2 + \dot{z}^2) + \frac{e}{c}(\dot{x}A_x + \dot{y}A_y + \dot{z}A_z) - V(x,z), \;\; \overrightarrow{A} = (0, 0, -xB) \tag{1}$$

or a non-relativistic 2D electronic system in a perpendicular magnetic field [1],

$$L = \sum_{\mu=1}^{N_e} \frac{1}{2}m_e \dot{\vec{x}}_\mu^2 - \frac{ieB}{2c}\varepsilon_{ij}x_\mu^i \dot{x}_\mu^j + V(\vec{x}_\mu) + \sum_{\mu<\nu} U(\vec{x}_\mu - \vec{x}_\nu), \tag{2}$$

the energy eigenvalues of the Landau Levels are:

$$E_n = \hbar\frac{eB}{m_e c}(n + \frac{1}{2}). \tag{3}$$

In the limit of the strong magnetic field, the separation between the Landau levels becomes very large and consequently only the LLL is relevant. One can neglect the kinetic term, i.e. formally put $m_e = 0$, the resulting Lagrangian is first order in time derivatives, turning the original coordinate space into an effective phase space defined by:

$$p_z \equiv \frac{\partial L_{LLL}}{\partial \dot{z}} = -\frac{eB}{c}x \;\Rightarrow\; \left[-\frac{eB}{c}x, z\right] = -i\hbar \;\Rightarrow\; [x, z] = i\frac{\hbar c}{eB} \equiv i\theta_L, \tag{4}$$

or

$$[x_\mu^i, x_\nu^j] = i\delta_{\mu\nu}\varepsilon^{ij}\frac{\hbar c}{eB} \equiv i\delta_{\mu\nu}\varepsilon^{ij}\theta_L. \tag{5}$$

The effects of truncation to the LLL are now expressed by noncommutativity, which is described by $\theta_L = \frac{\hbar c}{eB}$. It is essential that the equations (4,5) indicate that in the 3-dimensional space there is a noncommutative plane (NCP) perpendicular to the strong external magnetic field B.

The existence of NCP has been widely used to discuss the quantum Hall effect and relevant topics in condensed matter physics and mathematical physics [4, 5]. In such discussions on the quantum Hall effect, the noncommutative parameter for NCP is usually taken to be

$$\theta = f\theta_L, \tag{6}$$

where $f = f(\nu, B)$ is a function of the filling fraction ν and the magnetic field B, e.g. $f = \frac{1}{\nu} = \frac{eB}{2\pi\rho}$ in the noncommutative Chern-Simons theory [4], and it could be thought as an effective filling factor to characterize the possibility that the electrons stay on the LLL. At $f = 0$, no electron stays on the LLL, so that the NCP caused by the external magnetic field B is absent. For $f \neq 0$, the NCP exists and must be considered. In this paper $f(B)$ is treated as a phenomenology parameter.

A nature question arising from the condensed matter physics discussions mentioned above is whether such sort of NCP discussions can be extended into the QED dynamics of electron beam in accelerator, where the electrons are correlative to each other. It is always a possibility that some electrons stay on the LLL and $f \neq 0$, and there is no prior reason to ban this extension, hence the answer should be yes. As a matter of fact [7], the anomalous deviation of (g-2)-factor of muon to the prediction of the standard model has been attributed to the loop effects of QED with NCP. That could be thought as a rough estimation of the NCP effects in QED at loop level. However, the loop level process has some uncertainties both due to the theoretical treatment errors and the experimental measurement errors, and a tree level process in the accelerator experiments could be essential to make it clear. Hence, we consider the backward Compton scattering process in the strong magnetic field, e.g. the beamline BL38B2 in Spring-8, to explore whether the NCP effects exist or not.

The point for revealing the NCP effects caused by the LLL effect in a process is that the perpendicular external magnetic field B "felt" by the correlated electrons with *non-relativistic* motion should be very strong. As the backward Compton scattering is a process

that the soft laser photons are backscattered by the high energy electrons elastically, the motion of the electrons in the $e\gamma$-mass center frame (CM) is non-relativistic, the Lorentz factor to the laboratory frame is very large and the magnetic field "felt" by the electrons $B = B_{CM} = \gamma B_{Lab}$ becomes very large even if B_{Lab} is small. For instance, in the mass center frame of the beamline BL38B2 in Spring-8 with 8GeV electron, 0.01eV photon and 0.68T magnetic field, the velocity of the electron $v_{CM} \approx 0.0006 \ll 1,\ \gamma \approx 15645.6,\ B_{CM} \approx 10639T$. It well satisfies the precondition, hence the NCP due to the LLL could be described by a noncommutative quantum theory constructed in the mass center frame.

The contents of this paper are organized as follows: in Section II, we construct QED with NCP; in Section III, we derive the differential cross section of the backward Compton scattering process in a uniform perpendicular magnetic field; in Section IV, we produce the numerical results on it by using the data of Spring-8, and show how a precise measurement of the differential cross section leads to distinguishing the prediction of QED with NCP from the prediction of QED without NCP; finally, we briefly discuss the results.

II. QED WITH NCP

In order to construct the effective Lagrangian describing the electrons in the external magnetic field, the LLL effect should be considered. For the electrons stay on the LLL, the effects of projection on the LLL could be expressed by noncommutativity (natural units $\hbar = c = 1$):

$$[\hat{x}_\mu, \hat{x}_\nu] = i\theta_{\mu\nu} = i\theta C_{\mu\nu}, \quad \theta = f\theta_L = f\frac{1}{eB}, \tag{7}$$

$$C_{\mu\nu} = \begin{pmatrix} 0 & c_{01} & c_{02} & c_{03} \\ -c_{01} & 0 & c_{12} & -c_{13} \\ -c_{02} & -c_{12} & 0 & c_{23} \\ -c_{03} & c_{13} & -c_{23} & 0 \end{pmatrix}. \tag{8}$$

The Lagrangian in a noncommutative theory is fully covariant under observer Lorentz transformations: rotations or boosts of the observer inertial frame leave the physics unchanged because both the field operators and $\theta_{\mu\nu}$ transform covariantly [8]. In this paper, we calculate in the mass center frame, in which the motion of the electron is non-relativistic and only θ_{ij} are nonzero, and finally boost the results to the laboratory frame to compare with the experiment. The direction of the external magnetic field is $\hat{y}$ in the laboratory

frame, by means of the Lorentz transformation, the electron feels an electric field along $-\hat{x}$ and a magnetic field along $\hat{y}$ in the mass center frame. The electric field has no influence on the noncommutativity caused by the LLL [5], so that $c_{0i} = 0$. The magnetic field is along $\hat{y}$ and the NCP takes $(x,\ z)$-plane, so that $c_{13} = 1$ and other $c_{ij} = 0$.

Generally [2], we can implement the noncommutativity of space into path integral formulation through the Weyl-Moyal correspondence, and the noncommutative version of a field theory can be obtained by replacing the product of the fields appearing in the action by the star product:

$$(f * g)(x) = \lim_{\xi,\eta\to 0} \left[e^{\frac{i}{2}\partial_\xi^\mu \theta_{\mu\nu} \partial_\eta^\nu} f(x+\xi) g(x+\eta) \right]. \tag{9}$$

Following the general argument, we argue that the effective Lagrangian of QED with NCP (QED-NCP) for the electrons with $f(B) \neq 0$ should be an extension of the Lagrangian of NCQED with $f(B)$:

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} * F^{\mu\nu} + \overline{\psi} * (i\gamma^\mu D_\mu - m) * \psi, \tag{10}$$

with

$$D_\mu = \partial_\mu - ieA_\mu, \quad F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - ie[A_\mu, A_\nu]_*. \tag{11}$$

The above Lagrangian is invariant under the noncommutative U(1) transformation:

$$\begin{aligned}
&A_\mu \to A'_\mu(x) = U(x) * A_\mu * U(x)^{-1} + iU(x) * \partial_\mu U(x)^{-1}, \\
&F_{\mu\nu} \to F'_{\mu\nu} = U(x) * F_{\mu\nu} * U(x)^{-1}, \\
&\Psi(x) \to \Psi'(x) = U(x) * \Psi(x), \\
&U(x) = \exp *(i\lambda(x)) \equiv 1 + i\lambda(x) - \frac{1}{2}\lambda(x) * \lambda(x) + o(\theta^2).
\end{aligned}$$

Note that when $f(B) \to 0$, the Lagrangian of QED-NCP goes back to the ordinary QED Lagrangian. When $f(B) \ll 1$, the deviation of QED-NCP from QED can be calculated in perturbation, but no vacuum phase transition takes place. When B is extremely large (e.g. $\sim 10^9 T$), $f(B) \sim 1$ and the dynamical symmetry breaking may occur [3].

III. BACKWARD COMPTON SCATTERING

From the Lagrangian Eq.(10), the Feynman rules of QED-NCP can be obtained. The propagators of electron and photon remain unchanged, the vertices in QED-NCP (see Fig.1)

pick up additional kinematic phases from the Fourier transformation of new interactions. When the inverse Compton scattering by external electromagnetic fields or the synchrotron radiation is investigated, the A_μ in the Lagrangian of QED-NCP should be replaced by $A_\mu + A_\mu^{external}$. In this paper we do not study those processes, but only interest in the Compton scattering process, hence the $A_\mu^{external}$ and the four photon vertex are neglected.

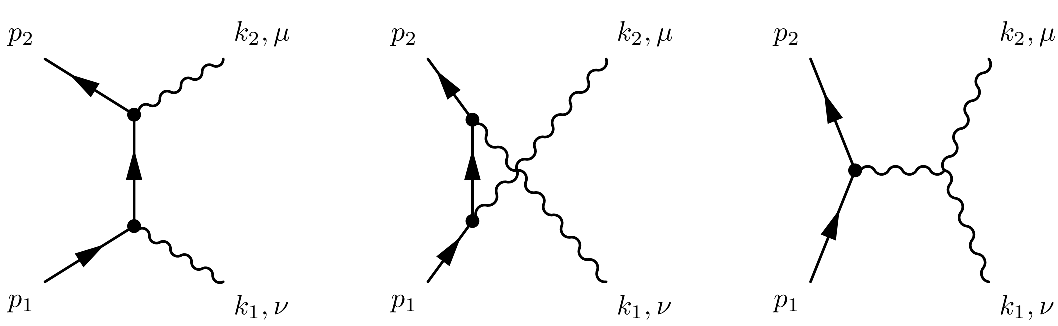
(a)$ie\gamma^\mu \exp(ip_1\theta p_2/2)$

(b)$2e\sin(k_1\theta k_2/2)((k_1-k_2)^\rho g^{\mu\nu} + (k_2 - k_3)^\nu g^{\rho\mu} + (k_3-k_1)^\mu g^{\nu\rho})$

FIG. 1: Feynman rules

Similar to the existed calculations of Compton scattering in NCQED [9], the Feynman diagrams, kinematics and the differential scattering cross section for the backward Compton scattering process in QED-NCP are as follows:

1. The Feynman diagrams of $e\gamma$-Compton scattering in QED-NCP are shown in Fig.2. $\mathcal{A}_i$ with $i = 1, 2, 3$ denote the amplitudes of corresponding diagrams. Compared with that in QED, there is an additional diagram $\mathcal{A}_3$ (see Fig.2(c)).



(a)$\mathcal{A}_1$ (b)$\mathcal{A}_2$ (c)$\mathcal{A}_3$

FIG. 2: Feynman diagrams

2. Kinematics (see Fig.3):

(a)The laboratory frame (b)The mass center frame

FIG. 3: Kinematics

i) The energies and momenta in the mass center frame:

$$
\begin{aligned}
& s=(p_1+k_1)^2, \quad t=(p_1-p_2)^2, \quad u=(p_1-k_2)^2, \\
& p_1=(\frac{s+m^2}{2\sqrt{s}},0,0,\frac{s-m^2}{2\sqrt{s}}), \;\; k_1=\frac{s-m^2}{2\sqrt{s}}(1,0,0,-1), \\
& p_2=\frac{s-m^2}{2\sqrt{s}}(\frac{s+m^2}{s-m^2},-\sin\vartheta\cos\phi,-\sin\vartheta\sin\phi,-\cos\vartheta), \\
& k_2=\frac{s-m^2}{2\sqrt{s}}(1,\sin\vartheta\cos\phi,\sin\vartheta\sin\phi,\cos\vartheta).
\end{aligned}
$$

ii) Polarization: We are interested in the process with polarized initial electrons, unpolarized or α-polarized initial photons (α is the angle between the magnetic field and the initial photon polarization), unpolarized final electrons and unpolarized final photons. So the following notations and formulas will be useful for our goal:

$$
\begin{aligned}
& \text{1) initial electron}: \;\; u_{-1/2}(p_1)\bar{u}_{-1/2}(p_1) \to \rho = \frac{1}{2}(\not{p}_1+m)(1-\gamma^5(-1)\gamma^2) \\
& \text{2) final electron}: \;\; \sum_i u_i(p_2)\bar{u}_i(p_2) \to \rho' = \not{p}_2+m \\
& \text{3) initial photon}: \;\; \frac{1}{2}\sum_i \epsilon^T_{i\mu}(k_1)\epsilon^{T*}_{i\mu'}(k_1) \to \xi_{\mu\mu'} \\
& \textit{or } \alpha-\text{polarized}: \;\; \epsilon^T_{\alpha\mu}(k_1)\epsilon^{T*}_{\alpha\mu'}(k_1) \to \xi_{\mu\mu'}, \;\; \epsilon^T_{\alpha\mu}=(0,\sin\alpha,\cos\alpha,0) \\
& \text{4) final photon}: \;\; \sum_i \epsilon^T_{i\nu}(k_2)\epsilon^{T*}_{i\nu'}(k_2) \to \xi'_{\nu\nu'}
\end{aligned}
$$

3. The differential cross section for the backward Compton scattering in QED-NCP is

$$
\frac{d\sigma}{d\phi d\cos\vartheta} = \frac{e^4}{64\pi^2 s}\xi_{\mu\mu'}\xi'_{\nu\nu'}Tr(\rho'\mathcal{A}^{\mu\nu}\rho\bar{\mathcal{A}}^{\nu'\mu'}), \tag{12}
$$

where $\mathcal{A}^{\mu\nu}=\mathcal{A}_1^{\mu\nu}+\mathcal{A}_2^{\mu\nu}+\mathcal{A}_3^{\mu\nu}$ and $\mathcal{A}_i^{\mu\nu}$, $\bar{\mathcal{A}}_i^{\nu'\mu'}$ $(i=1,2,3)$ are:

$$
\mathcal{A}_1^{\mu\nu} \;=\; (-1)e^{ip_1\theta p_2/2}e^{ik_1\theta p_2/2}\gamma^\mu\frac{\not{p}_1+\not{k}_1+m}{(p_1+k_1)^2-m^2}\gamma^\nu
$$

$$
\begin{aligned}
\mathcal{A}_2^{\mu\nu} &= (-1)e^{ip_1\theta p_2/2}e^{-ik_1\theta p_2/2}\gamma^{\nu}\frac{\not{p}_1-\not{k}_2+m}{(p_1-k_2)^2-m^2}\gamma^{\mu} \\
\mathcal{A}_3^{\mu\nu} &= (-i)e^{ip_1\theta p_2/2}2\sin(k_1\theta k_2/2)\gamma^{\sigma}[g_{\rho\sigma}/(k_1-k_2)^2] \\
&\quad [(k_1+k_2)^{\rho}g^{\mu\nu}+(k_1-2k_2)^{\nu}g^{\rho\mu}+(k_2-2k_1)^{\mu}g^{\nu\rho}] \\
\bar{\mathcal{A}}_1^{\nu'\mu'} &= (-1)e^{-ip_1\theta p_2/2}e^{-ik_1\theta p_2/2}\gamma^{\nu'}\frac{\not{p}_1+\not{k}_1+m}{(p_1+k_1)^2-m^2}\gamma^{\mu'} \\
\bar{\mathcal{A}}_2^{\nu'\mu'} &= (-1)e^{-ip_1\theta p_2/2}e^{+ik_1\theta p_2/2}\gamma^{\mu'}\frac{\not{p}_1-\not{k}_2+m}{(p_1-k_2)^2-m^2}\gamma^{\nu'} \\
\bar{\mathcal{A}}_3^{\nu'\mu'} &= (i)e^{-ip_1\theta p_2/2}2\sin(k_1\theta k_2/2)\gamma^{\sigma'}[g_{\rho'\sigma'}/(k_1-k_2)^2] \\
&\quad [(k_1+k_2)^{\rho'}g^{\mu'\nu'}+(k_1-2k_2)^{\nu'}g^{\rho'\mu'}+(k_2-2k_1)^{\mu'}g^{\nu'\rho'}]
\end{aligned}
$$

We define the phase factor $\Delta \equiv \frac{k_1\theta p_2}{2} = -\frac{k_1\theta k_2}{2} = \frac{f(s-m^2)^2}{8Bes}\sin\vartheta\cos\phi$ (notation $k\theta p \equiv k^{\mu}\theta_{\mu\nu}p^{\nu}$), and then the differential cross sections of the backward Compton scattering with polarized initial electrons, unpolarized initial photons, unpolarized final electrons and unpolarized final photons in QED-NCP are:

$$
\begin{aligned}
\frac{d\sigma}{d\phi d\cos\vartheta} &= \frac{e^4}{32\pi^2 s}\left((s-m^2)^2+(u-m^2)^2-\frac{4m^2t(m^4-su)}{(s-m^2)(u-m^2)}\right) \\
&\quad \times\left(-\frac{1}{(s-m^2)(u-m^2)}+\frac{4\sin^2\Delta}{t^2}\right).
\end{aligned} \tag{13}
$$

Note that it's $f(B)$ dependent and goes back to that in QED as $f(B)\to 0$, and coincides with that in NCQED [9] as $m\to 0$. Similarly, for the processes with any polarization, the differential cross sections could be calculated, some numerical results are as follows.

IV. NUMERICAL RESULTS

In this section, the data of BL38B2 in Spring-8 will be used to discuss the QED-NCP predictions of backward Compton scattering numerically. The accelerator diagnosis beamline BL38B2 in Spring-8 has a bending magnet light source, $10MeV$ γ-ray photons are produced in the magnetic field by the backward Compton scattering of far-infrared (FIR) laser photons. The energy of electron in the storage ring is $8GeV$, the perimeter of the ring is $1436m$, the wavelength of FIR laser photon is $119\mu m$ and the magnetic field is $0.68T$. Then, in the mass center frame, the Lorentz factor $\gamma \approx 15645.6$, the magnetic field is $2\times 10^6 eV^2 \approx 10639T$ (hence the LLL effect is relevant), θ_L is $1.6\times 10^{-6}eV^{-2}\approx (2.5\text{Å})^2$ and the phase

factor becomes $\Delta \approx 0.0844 f \sin\vartheta \cos\phi$. Substituting all of these into Eq.(12), the realistic calculations are doable. Fig.4 shows a measurement of the differential cross section to final photon energy of the backward Compton scattering in Spring-8, in order to compare with it, the ϕ-integrated energy dependence of the differential cross section is calculated.

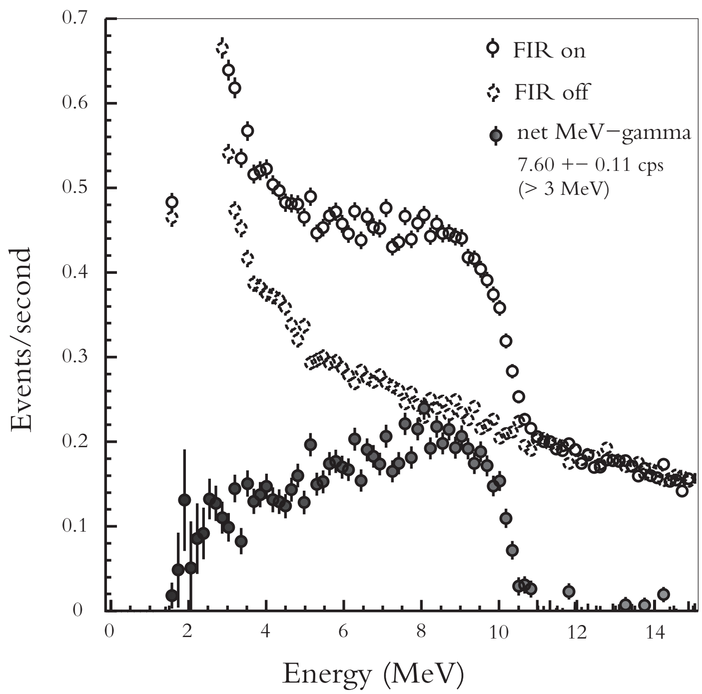


FIG. 4: Spring-8 data for $e\gamma \rightarrow e'\gamma'$ [10]. The γ-ray spectrum from the backward Compton scattering process has been deduced after the subtraction of the "FIR laser off" spectrum from the "FIR laser on" spectrum. They are shown by the solid circles and proportional to $\frac{d\sigma(E_\gamma)}{dE_\gamma}$.

Suppose the initial photon is unpolarized, from Fig.4, we can roughly see:

$$\mathcal{R}|_{expt} = \frac{d\sigma(5MeV)/dE_\gamma}{d\sigma(9MeV)/dE_\gamma}|_{expt} \approx \frac{0.15}{0.22} \approx 0.68. \tag{14}$$

However, we find out that $\mathcal{R}|_{expt}$ is significantly larger than the QED prediction (Fig.5(a)):

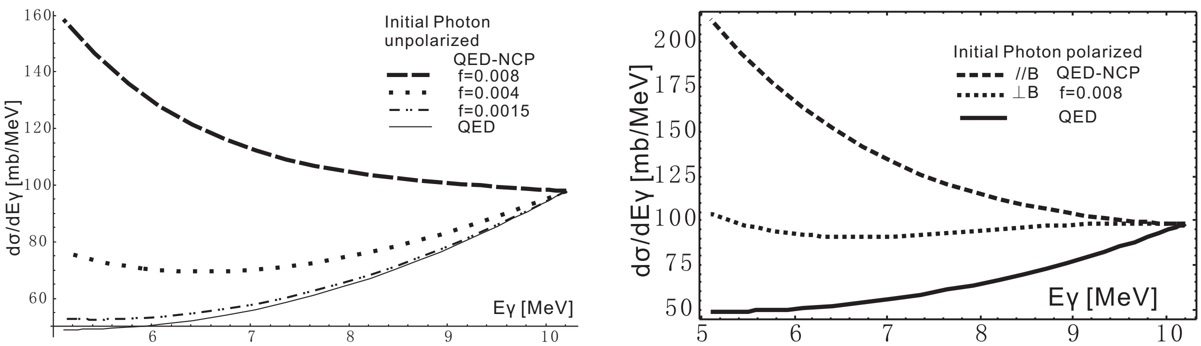
$$\mathcal{R}|_{QED} = \frac{d\sigma(5MeV)/dE_\gamma}{d\sigma(9MeV)/dE_\gamma}|_{QED} \approx \frac{48.87}{77.43} \approx 0.63. \tag{15}$$

A natural interpretation to this deviation is that the possibility that the electrons stay on LLL is nonzero, and there is a NCP in the external magnetic field, which hasn't been taken into account in QED. By means of QED-NCP, and adjusting the effective filling factor $f(B)$,

a suitable $\mathcal{R}|_{QED-NCP}$ consistent with $\mathcal{R}|_{expt}$ can be obtained. The corresponding prediction with $f(B) = 0.0015$ is shown in Fig.5(a):

$$\mathcal{R}|_{QED-NCP} = \frac{d\sigma(5MeV)/dE_\gamma}{d\sigma(9MeV)/dE_\gamma}|_{QED-NCP} \approx \frac{52.88}{78.24} \approx 0.68. \tag{16}$$

However, photon polarization, detector inefficiency and radiation corrections due to mirror and windows will all affect the shapes of experimental data, the uncertainties of current experimental data are too large to separate two calculations. It is still too early to decide the existence of the NCP effects. A further precise measurement is needed.



(a)Unpolarized initial photon

(b)$\hat{x}, \hat{y}$-polarized initial photon

FIG. 5: Energy dependence of the differential cross section.

Theoretically, for a 2D electronic system, $f = \frac{eB}{2\pi\rho}$ proved in [4] can be used. The electron beam of BL38B2 in Spring-8, whose charge is around $1.44nC$, length is $13ps$, horizontal size is $114\mu m$ and vertical size is $14\mu m$ ($\ll$ horizontal size), is a near 2D electronic system. Hence, a rough prediction of $f(B)$ could be calculated:

$$\rho \approx \frac{1.44nC/(1.6 \times 10^{-19}C)}{13ps \times (3 \times 10^8 m/s) \times 114\mu m} \approx 2 \times 10^{12} cm^{-2}, \tag{17}$$

$$f \approx \frac{0.68T \times (2 \times 0.511MeV \times 5.788 \times 10^{-11} MeV/T)}{2\pi \times 2 \times 10^{12} cm^{-2} \times (197.3MeV \times fm)^2} \approx 0.008. \tag{18}$$

With a typical $f(B) = 0.008$, we further consider experiments with polarized initial photon. The initial laser photons move along the direction of $\hat{z}$ and their polarization is taken either parallel or perpendicular to the magnetic field direction of $\hat{y}$. As shown in Fig.5(b), the energy dependence of the differential cross sections with the $\hat{x}$-polarized ($\perp B$) and the $\hat{y}$-polarized ($\| B$) initial photons are the same in QED, and different in QED-NCP. This strongly suggests that a precise backward Compton scattering experiment in Spring-8

with differently polarized initial photons is most favorable for testing the NCP effects. The experiment with different initial photon polarization is practicable to reveal the NCP effects, because the subtraction of the $\perp B$-polarized spectrum from the $\| B$-polarized spectrum can reduce the experimental uncertainties.

Furthermore, we consider the total cross sections (barn) by integrating E_γ from $5.1MeV$ to $10.2MeV$ (or integrating ϑ from 0 to $\pi/2$):

$$\begin{aligned}
\sigma_{QED} = \sigma_{QED}^{\perp B} = \sigma_{QED}^{\| B} \approx 0.586936, &\quad \sigma_{QED-NCP}^{\perp B} \approx 0.586936 + 2828.44 f^2, \\
\sigma_{QED-NCP} \approx 0.586936 + 4384.20 f^2, &\quad \sigma_{QED-NCP}^{\| B} \approx 0.586936 + 5939.96 f^2, \\
\sigma_{QED-NCP} - \sigma_{QED} \approx 7469.64 f^2 \sigma_{QED}, &\quad \sigma_{QED-NCP}^{\| B} - \sigma_{QED-NCP}^{\perp B} \approx 5301.30 f^2 \sigma_{QED}.
\end{aligned}$$

From above we can see that the difference between the total cross sections of QED and QED-NCP is proportional to f^2, and the difference between the total cross sections with the $\perp B$-polarized initial photons and with the $\| B$-polarized ones is proportional to f^2, too, hence $f(B)$ characterizing the NCP effects could also be determined in the $e\gamma$-total cross section measurements.

V. SUMMARY AND DISCUSSION

In this paper, the NCP caused by the LLL effect in the strong uniform perpendicular magnetic field, and QED with NCP are studied. For the process of backward Compton scattering in the magnetic field of the storage ring magnet in accelerator, the amplitudes and the differential cross sections in QED-NCP are calculated. Numerical predictions of the energy dependence of the differential cross sections in QED-NCP and in QED are calculated with the parameters of BL38B2 in Spring-8, and compared with the existing data of BL38B2. It indicates that a precise measurement of the energy dependence of the differential cross sections of backward Compton scattering with polarized photon in a strong perpendicular magnetic field would be practicable to distinguish the prediction of QED with NCP from the prediction of QED without NCP and may reveal the effects of NCP. Such an experiment is expected.

Being similar to the noncommutative Chern-Simons theory of the fractional quantum Hall effect, an effective filling factor $f(B)$ is introduced to characterize the possibility that the electrons stay on the LLL. In this paper $f(B)$ is treated as a phenomenology parameter

and expected to be determined experimentally. A further task is to estimate it theoretically. In Section IV, we present a rough estimation of it for BL38B2 in Spring-8. It seems to be reasonable for near 2D correlated electrons with non-relativistic motion in the external magnetic field, and supports the NCP discussion of backward Compton scattering in accelerator. However, the equation (17) is a rough approximative estimation of the 2D electron density under the assumption that the electron beam is evenly distributed in a finite 2D rectangle, i.e., $\rho(x,z)|_{(x,z)\in\ rectangle} = constant$. In a real beamline, however, the 2D density should be electron-distribution dependent, e.g., with a Gaussian distribution, we may need to correct the density ρ in Eq.(17) with a factor α, i.e., $\rho \to \alpha\rho$, where $\alpha = 1/2\pi$ or $1/4\pi$. In this case, the numerical results of $d\sigma/dE_\gamma$ in Fig.5 will receive a correction from α. We argue that this correction would not lead to the change of the basic scenario of $d\sigma/dE_\gamma$ due to QED-NCP. The discussions in Section IV are instructive, but a more sound theoretical study on $f(B)$ for the electrons in accelerator is still wanted, and a detailed discussion on the effects of NCP remains to be further explored.

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